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RM-6463

B. E. - II (Sem. IV) (Comp./IT) Examination

May / June – 2010

Engineering Mathematics - III

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. - 2 (Sem. 4) (Comp./IT)**

Name of the Subject :
☛ **Engineering Mathematics - 3**

☛ Subject Code No. : **6 4 6 3** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) Attempt **all** questions.
(3) Figures to right indicate the maximum marks of the question.
(4) Write each section in **separate** answer books.

SECTION - I

1 (a) Do as directed : **10**

(1) Change the order of integration :

$$I = \int_0^1 \int_{x^2}^{2-x} xy\, dA$$

(2) Change into cylindrical co-ordinates :

$$I = \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 dz\, dy\, dx$$

(3) Prove that : $\nabla r^2 = 2\,\bar{r}$

(4) If $\bar{F}$ is conservative, then $\bar{F}$ is irrotational.

(5) Find $f(0)$ for
$$f(x) = 1+\frac{2x}{\pi}, \quad -\pi \le x \le 0$$
$$= 1-\frac{2x}{\pi}, \quad 0 \le x \le \pi$$

(b) Attempt any **three** : **12**

(1) Evaluate $\iint_R \left(x^2+y^2\right)x\, dx\, dy$ over the positive quadrant of the circle $x^2+y^2=a^2$.

(2) Evaluate $\iiint_D \frac{dv}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$ where D is the region bounded by the spheres $x^2+y^2+z^2=a^2$ and $x^2+y^2+z^2=b^2$ $(a>b>0)$

(3) Find the area that lies inside the cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$.

(4) Find the volume bounded by the xy-plane, the cylinder $x^2+y^2=1$ and the plane $x+y+z=3$.

(5) Find the surface area of the section of the cylinder $x^2+y^2=a^2$ made by the plane $x+y+z=a$.

2 (a) Attempt any **two** : **6**

(1) Find the directional derivative of $\phi=3e^{2x-y+z}$ at $A(1, 1, -1)$ in the direction $\overrightarrow{AB}$ where B is the point $(-3, 5, 6)$.

(2) Prove that $r^n . \bar{r}$ is irrotational and is solenoidal when $n=-3$.

(3) A vector field is given by $\bar{F}=\left(x^2+xy^2\right)i+\left(y^2+x^2y\right)j$ Show that $\bar{F}$ is irrotational and find its scalar potential.

(b) Attempt any **two** : **8**

(1) Verify Green's theorem for the function $\bar{F}=\left(x^2+y^2\right)i-2xy\, j$ and C is the rectangle in the xy-plane bounded by $y=0$, $y=b$, $x=0$ and $x=a$.

(2) Use divergence theorem to evaluate $\iint_S \left(x^3dydz+x^2ydz\, dx+x^2zdxdz\right)$ where S is the closed surface consisting of the cylinder $x^2+y^2=a^2$ and the circular discs $z=0$ and $z=b$.

(3) Apply Stoke's theorem for $\bar{F} = \left(x^2 - y^2\right)i + 2xyj$ in the rectangular region $x = 0$, $y = 0$, $x = a$, $y = b$.

3 (a) Explain half range Fourier series. **4**

(b) Attempt any **two** : **10**

(1) Expand $f(x) = \sin ax$, $-\pi < x < \pi$ in Fourier series.

(2) Obtain Fourier expansion for the function $f(x)$ with period given by

$$f(x) = -x^2 \quad \text{if} \quad -\pi < x < 0$$
$$= x^2 \quad \text{if} \quad 0 < x < \pi$$

(3) Express $\sin x$ as cosine series in $0 < x < \pi$.

SECTION - II

4 (a) Attempt the following : **10**

(1) Show that Beta function is symmetric.

(2) State one dimensional wave equation and express its solution in terms of Fourier coefficients.

(3) Define Gamma function and hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(4) Show that any solution of the Laplace's equation, $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ is called a harmonic function.

(5) Derive the Cauchy-Riemann equations in polar coordinates.

(b) Attempt any **two** : **6**

(1) Prove that $\int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx = \Gamma(n)$

(2) Evaluate $\int_0^\infty \frac{x^8\left(1 - x^6\right)}{(1+x)^{24}} dx$

(3) Prove that $\int_0^\infty e^{-(x+a)^2} dx = \frac{\sqrt{\pi}}{2}\left[1 = erf(a)\right]$

(c) Solve any **two** : **6**

(1) $x^2 p + y^2 q = (x + y) z$ (2) $pz - qz = z^2 + (x + y)^2$

(3) $\frac{y^2 z}{x} p + xzq = y^2$

5 Attempt any **two** : 14

(1) Determine the solution of the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ using the variable separable method.

(2) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly coold to 0° C and are kept at that temperature function $u(x, t)$.

(3) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions :

$u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin \frac{n\pi x}{l}$

6 (a) Attempt any **two** : 8

(1) Consider the transformation $w = e^{i\frac{\pi}{4}} z$ and determine the region in the w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane.

(2) An electrostatic field in the xy-plane is given by the potential function $\phi = 3x^2 y - y^3$, find the stream function.

(3) Determine the region of the w-plane into which the region $\frac{1}{2} \le x \le 1$ and $\frac{1}{2} \le y \le 1$ is mapped by the transformation $w = z^2$.

(b) Attempt any **two** : 6

(1) Evaluate $\int_0^{1+i} \left(x - y + ix^2\right) dz$ along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis form $z = 1$ to $z = 1 + i$,

(2) Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$ along the line $2y = x$.

(3) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$.